# Predictability in the Shape of the Term Structure of Interest Rates

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We know a considerable amount about the performance of tactical style allocation models in equity markets, but very little evidence is available on the performance of systematic dynamic allocation decisions on various bond benchmarks with different maturities. Most of the literature on predictability in bond returns focuses on timing bonds versus stocks or bonds versus cash, with no emphasis on the timing of bonds with different maturities.

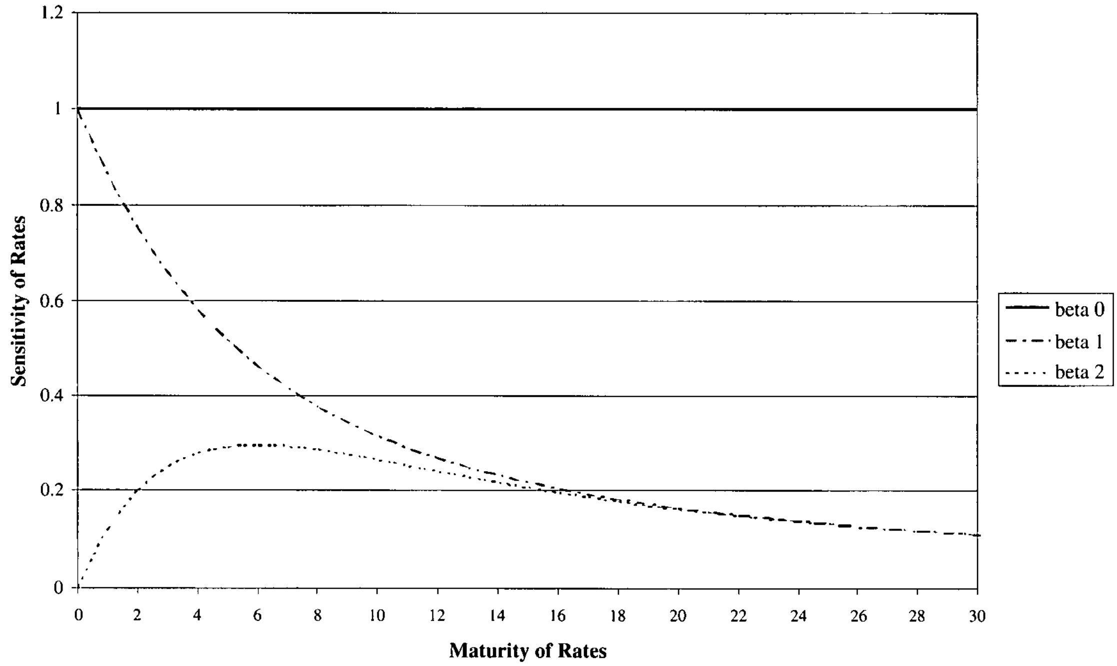
Research on tactical asset allocation decisions involving bond markets includes Shiller [1979], Fama [1981], Shiller, Campbell, and Schoenholtz [1983], Keim and Stambaugh [1986], Campbell [1987], Fama and Bliss [1987], Fama and French [1989], Campbell and Shiller [1991], Ilmanen [1995, 1997], Bekaert, Hodrick, and Marshall [1997], Lekkos and Milas [2001], Ilmanen and Sayood [2002], and Baker, Greenwood, and Wurgler [2003]. These authors focus on exploiting predictability in a global bond portfolio and hence in the *level* of interest rates, but they do not try to exploit predictability on other dimensions of the shape of the yield curve such as *slope* and *curvature*.[1]

It is only recently that some articles have recognized the benefits of exploiting predictability in the shape of the yield curve, although to the best of our knowledge, there are only two. Dolan [1999] argues that the curvature parameter of the yield curve, estimated using the Nelson-Siegel [1987] model, can be predicted using simple parsimonious models, and shows these forecasts have investment significance in the selection of bullet over barbell portfolios. Diebold and Li [2002] estimate autoregressive models for predicting Nelson-Siegel level, slope, and curvature factors.

We extend this research on several dimensions. First, we test for statistical significance in the predictive power of a series of economically meaningful variables. This approach stands in sharp contrast with Dolan [1999] and Diebold and Li [2002], who use only information about past values of the term structure parameters in their predictive experiments. We thus bridge the gap in the literature on predictability of asset returns on the basis of variables such as dividend yields or term spread. Our work is also related to research based on joint macrofinance modeling strategy of the term structure of interest rates (e.g., Ang and Piazzesi [2003], Diebold, Rudebusch, and Aruoba [2005], or Rudebusch and Wu [2004]).

Like Pesaran and Timmermann [1995], we investigate the predictability of bond portfolio returns using a robust recursive modeling approach based on multifactor models. This allows us to alleviate concerns over spurious results driven by data-mining biases. In the interest of robustness and in an attempt to account for model uncertainty, we use a Bayesian econometric approach, known as *thick* modeling, which selects at each date a "council" of models to make predictions rather than a single model.

## EXHIBIT 1
**Sensitivity of Zero-Coupon Rates to Parameters**




Another contribution is demonstration of how this predictability in various segments of the yield curve can be used to generate significant outperformance through systematic trading strategies involving simple bullet and barbell bond portfolios and butterfly strategies with fixed-income derivatives.

## I. DATA AND METHODOLOGY

Following Dolan [1999] and Diebold and Li [2002], we use a parsimonious model of the yield curve to extract the time-varying parameters that we adopt as a proxy for factors affecting the shape of the yield curve. We use the Nelson and Siegel [1987] model. One could also use the Vasicek [1977] model or the extended Vasicek model, among many others.[2]

The Nelson-Siegel model has become a popular way for practitioners to parameterize the term structure of interest rates. Consistent with principal components analysis (PCA) results, it entails four parameters and is modeled as follows:[3]

$$R(t,\theta) = \beta_0 + \beta_1\left[\frac{1-\exp(-\theta/\tau)}{\theta/\tau}\right] + \beta_2\left[\frac{1-\exp(-\theta/\tau)}{\theta/\tau} - \exp(-\theta/\tau)\right] \quad (1)$$

where:

$R(t, \theta)$ = rate at time zero with maturity $\theta$;
$\beta_0$ = limit of $R(t, \theta)$ as $\theta$ goes to infinity. In practice, $\beta_0$ should be regarded as a long-term interest rate;
$\beta_1$ = limit of $\beta_0 - R(t, \theta)$ as $\theta$ goes to 0. In practice, $\beta_1$ should be regarded as the short- to long-term spread;
$\tau$ = scale parameter that measures the rate at which the short-term and medium-term components decay to zero; and
$\beta_2$ = curvature parameter.

The advantage of this model is that the three parameters $\beta_0$, $\beta_1$, and $\beta_2$ can directly be interpreted as level, slope, and curvature changes in the yield curve. As illustrated in Exhibit 1, the sensitivities $S_i = \partial R(t, \theta)/\partial \beta_i$ of swap rates to each parameter $\beta_i$ for $i = 0, 1, 2$ can be interpreted as follows. The level factor $S_0$ is constant across maturities. The slope factor $S_1$ is highest for short maturities and declines exponentially toward zero as maturity increases. Starting from zero for short maturities, the curvature factor $S_2$ reaches a maximum at the middle of the maturity spectrum and then declines to zero for longer maturities.

The parameters $\beta_0$, $\beta_1$, and $\beta_2$ are estimated monthly using an ordinary least squares optimization program, which consists, for a basket of yields, of minimizing the sum of the squared spread between the market yield and the theoretical yield obtained with the model.

We see that the evolution of the swap rate $R(t, \theta)$ is driven entirely by evolution of the beta parameters, as the scale parameter $\tau$ is fixed and taken to equal 3.[4] More specifically, we estimated the parameters $\beta_0$, $\beta_1$, and $\beta_2$ as follows. We use monthly data from June 7, 1994, through September 5, 2003, on 12 yields: 3-month, 6-month, and 1-, 2-, 3-, 4-, 5-, 7-, 10-, 15-, 20-, and 30-year swap rates.[5]

Each month, we estimate the swap curve by minimizing the ordinary least squares model:

$$\underset{\beta_0,\beta_1,\beta_2}{Min} \sum_{i=1}^{13} \left[ \begin{array}{l} S(t,\theta_i) - \beta_0 - \beta_1\left(\dfrac{1-\exp(-\theta_i/\tau)}{\theta_i/\tau}\right) - \\ \beta_2\left(\dfrac{1-\exp(-\theta_i/\tau)}{\theta_i/\tau} - \exp(-\theta_i/\tau)\right) \end{array} \right]^2 \quad (2)$$

where:

$S(t, \theta_i)$ = actual market yield with maturity $\theta_i$, and
$R(t, \theta_i)$ = theoretical yield with maturity $\theta_i$ given by the model [see Equation (1)].

We try to predict the dynamics of changes in beta parameters, regarded respectively as changes in level, slope, and curvature coefficients. Exhibit 2 plots the time evolution of these parameters in the sample period, and Exhibit 3 reports some basic descriptive statistics.

A casual inspection of Exhibits 2 and 3 seems to suggest that $\beta_0$ and $\beta_1$ are not stationary, while $\beta_2$ seems to be. To confirm this first impression, we perform formal Dickey-Füller tests of stationarity. The null hypothesis of a unit root in the dynamics of the beta parameters is rejected only in the case of $\beta_2$. Yet after differentiating once, all three parameters appear to be stationary. In other words, while $\beta_0$ and $\beta_1$ are I(0) processes, $\beta_2$ is an I(1) process.

Further analysis shows little or no evidence of the presence of autocorrelation in the differentiated series, which suggests that autoregressive models based only on past data are not likely to have good predictive power. $\beta_0$ in particular seems to follow a random walk, and we expect it will be hard to predict change in this parameter.

It is only in the case of changes in $\beta_2$ that an AR(1) model can be fitted to the data. The results for the period covering July 1994 to September 2003 show an R-square of 0.747, with a t-statistic of 18.654 associated with the lagged value of changes in $\beta_2$, which therefore appears strongly significant. The model, however, delivers only modest out-of-sample performance in terms of predictive ability.

Overall, this suggests one should try to use lagged explanatory variables to predict changes in the level, slope, and curvature coefficients, rather than only past values of these parameters as in Dolan [1999] and Diebold and Li [2002].

We first consider, for the sake of illustration, a simple vector autoregressive (VAR) model for the dynamics of $\beta_1$:

$$\begin{aligned} d\beta_1(t) = \alpha d\beta_0(t-1) + \gamma d\beta_0(t-2) + \\ \delta d\beta_1(t-2) + \varepsilon_t \end{aligned} \quad (3)$$

where $d\beta_i(t) := \beta_i(t) - \beta_i(t-1)$, and $\varepsilon_t$ is a white noise process. This model has the highest explanatory power measured in terms of the Schwartz information criterion for the calibration sample (September 1994 through August 1998).

The model is calibrated on a four-year rolling window sample, and used to make out-of-sample predictions for the period from September 1998 to September 2003. Exhibit 4 provides information on the estimated model for the last sample date (calibration sample covering September 1999 through August 2003, for predicting the value of the change in $\beta_1$ between August 2003 and September 2003). We use White heteroscedasticity-consistent standard errors and covariance estimates.

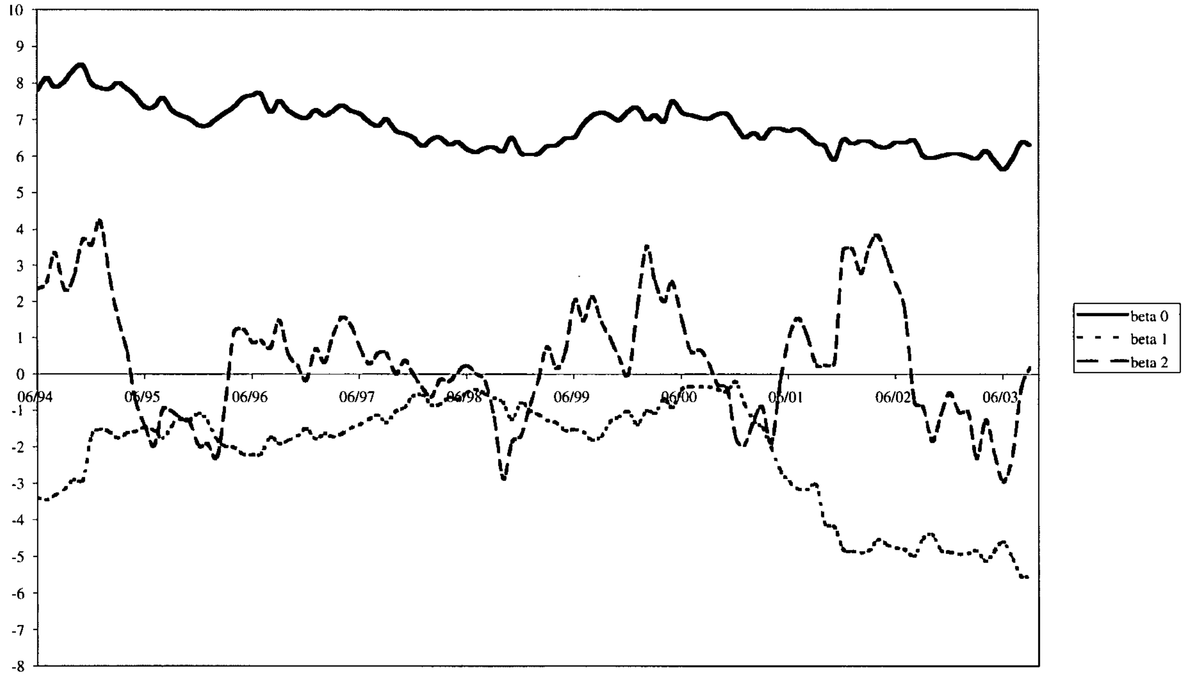
The R-square for the model is 0.326. For out-of-sample prediction purposes in the 61 months from September 1998 to September 2003, the model generates a hit rate of 62%, statistically greater than 50% at the 5% level.

We also try to test the robustness of the forecast as a function of confidence in the prediction as follows. Assuming that the error in prediction is normally distributed, we can estimate the probability of predicting that $\beta_1$ will increase (decline) while it will actually decline (increase), and we may want to make predictions only when confidence is high.

We define x% as a confidence level as follows. When the probability of a correct forecast is lower than 50% – x%, we do not make any prediction. The resulting forecasting success as a function of x is presented in Exhibit 5.

# EXHIBIT 2

**Time Series of Beta Parameters in Nelson-Siegel Model**




# EXHIBIT 3

**Beta Parameter Statistics**

| | Mean | St. Dev. | Serial Correlation | Correl. with beta 0 | Correl. with beta 1 | Correl. with beta 2 |
|---|---|---|---|---|---|---|
| Beta 0 | 6.848202 | 0.622541 | 0.94 | 1 | 0.321486 | 0.463554 |
| Beta 1 | -2.19569 | 1.545144 | 0.98 | 0.321486 | 1 | -0.10449 |
| Beta 2 | 0.40457 | 1.670274 | 0.86 | 0.463554 | -0.10449 | 1 |

The numbers reported show a regular increase in the hit rate as a function of x, which can be taken as an indication of robustness in the out-of-sample forecasting exercise.

This illustration suggests there is some degree of predictability in the time series evolution of beta parameters.

## II. FORECASTING CHANGES IN THE SHAPE OF THE YIELD CURVE

A more thorough analysis of the predictability of parameter evolution uses economically motivated variables as predictors. We first report some evidence of in-sample predictability in the shape of the yield curve, depending on a limited number of economically motivated variables. We then analyze out-of-sample predictability using multivariate models.

# EXHIBIT 4

**Information for Last Sample Date**

| Variable | Coefficient | Standard Deviation | T-Statistic | Prob. |
|---|---|---|---|---|
| $d(\beta_0)(-1)$ | 0.572229 | 0.215721 | 2.652632 | 0.0110 |
| $d(\beta_0)(-2)$ | 0.857365 | 0.191954 | 4.466515 | 0.0001 |
| $d(\beta_1)(-2)$ | 0.404483 | 0.106880 | 3.784466 | 0.0005 |

### In-Sample Evidence of Predictability

Trying to screen hundreds of variables using stepwise regression techniques usually leads to high in-sample R-squares but low out-of-sample R-squares (a robustness problem). Therefore, to forecast changes in beta parameters, we instead consider a short list of meaningful variables chosen on the basis of some evidence of their ability to predict asset returns, as well as their natural influence on asset returns.

## EXHIBIT 5

**Results of Forecasting Exercise**

| x | 0% | 5% | 10% | 15% | 20% | 25% | 30% |
|---|---|---|---|---|---|---|---|
| Number of Bets | 61 | 50 | 31 | 23 | 17 | 14 | 10 |
| Hit Rate | 62% | 62% | 74% | 78% | 82% | 86% | 90% |

Most of these variables fall into three broad categories.

1. Variables related to interest rates:
   - Level of the term structure of interest rates, proxied by the short-term rate. Fama and Schwert [1977] and Fama [1981] show this variable is negatively correlated with future stock market returns; it serves as a proxy for expectations of future economic activity.
   - Slope of the term structure of interest rates, proxied by the term spread. An upward-sloping yield curve signals expectations of an increase in the short-term rate, usually associated with an economic recovery.
   - Expectations of future values of interest rates, proxied by the mean one-year forward rates for maturities ranging from one to five years. It has been argued that forward rates can be used to predict future bond returns (see Fama and Bliss [1987] or Cochrane and Piazzesi [2002]).[7]
2. Variables related to risk:
   - Quantity of risk, proxied by historical volatility (intramonth volatility of stock returns) or expected volatility (implicit volatility from option prices).
   - Price of risk, proxied by credit spreads on high-yield debt as well as emerging market credit spreads. The price of risk captures the effect of default premiums, which track long-term business cycle conditions (higher during recessions, lower during expansions) (see Fama and French [1998]).
3. Variables related to relative cheapness of stock prices, proxied by dividend yields: It has been shown that the dividend yield is associated with slow mean reversion in stock returns across several economic cycles (Keim and Stambaugh [1986], Campbell and Shiller [1991], Fama and French [1998]). It serves as a proxy for time variation in the unobservable risk premium, since a high dividend yield indicates that dividends have been discounted at a higher rate.

We also include a short list of additional variables that are known to have a natural impact on the shape of the yield curve. The first one is the U.S. capacity utilization rate; when it is high, a given indication of economic growth is more likely to lead to inflation concerns, and therefore to potential increases in interest rates.[8]

We also include a *sentiment* variable, a measure of imbalance between market volume on puts versus calls such as the ratio of volume of call to volume of put options, and a measure of relative cheapness of the bond market versus the stock market through the differential between the E/P ratio on the S&P 500 and the yield of the ten-year Treasury note. We finally include as in Ilmanen [1995] a measure of U.S. inverse relative wealth as a proxy for time-varying risk aversion (because relative risk aversion is negatively related to relative wealth), which in turn can explain time variations in the risk premium.

Exhibit 6 lists the 12 variables and past values of changes in the beta parameters themselves. Monthly data on these variables are collected from DataStream (Thomson Financial) from September 1994 to September 2003. In a first-step analysis, we run in-sample first-pass regressions of changes in the beta parameters on these one-month lagged variables. Exhibit 6 provides information on the $t$-statistic associated with the slope coefficient of the regression and out-of-sample hit rates of predictive models based on such single-variable OLS regressions.

Overall, none of the selected variables (one-month lagged value) appears significant at the 5% level for predicting changes in the level of the yield curve, and hit rates for forecasts using a simple OLS regression are poor. These findings again strongly suggest that there is little predictability in long-term rates over a one-month horizon. Similar results are obtained for changes in the curvature component. The results are much better for changes in the slope of the yield curve, as several variables seem to have a significant lagged impact on changes in this variable, with relatively high levels of associated hit rates.

### Out-of-Sample Evidence of Predictability

While we obtain encouraging results for predictability of changes in the slope parameter, there are a number of reasons to go beyond such a simplified analysis. First, some of these variables may not show predictive power at a one-month lag but be more significant at a different lag. More important, a single-factor specification

## EXHIBIT 6
**List of Variables and Lagged Impact on Beta Parameters**

| Name | T-Stat | | | Hit Rate | | |
|---|---|---|---|---|---|---|
| | $d\beta_0$ | $d\beta_1$ | $d\beta_2$ | $d\beta_0$ | $d\beta_1$ | $d\beta_2$ |
| Default Spread | 0.93 | -1.87 | -0.11 | 33% | 63% | 58% |
| CBOE OEX Volatility (VIX) | -0.09 | -2.14 | 0.35 | 42% | 63% | 42% |
| Intramonth Volatility of Bond Returns | 0.37 | 1.85 | 1.14 | 54% | 63% | 46% |
| S&P 500 Dividend Yield | -0.61 | 1.58 | -0.73 | 58% | 63% | 50% |
| US Capacity Utilization Rate | -0.57 | 2.28 | -0.59 | 50% | 63% | 58% |
| Spread Emerging Market | 1.38 | -0.62 | 2.44 | 50% | 63% | 42% |
| Lehman U.S. Treasury Bills | 0.83 | -3.97 | 1.29 | 58% | 67% | 58% |
| E/P – Yield of 10-year T-Note | 0.59 | -3.54 | 1.66 | 54% | 50% | 63% |
| Term Spread (20 Year – T-Bill) | 0.19 | 0.81 | 0.41 | 38% | 58% | 46% |
| Mean U.S. Forward Rate | -1.16 | 2.57 | -1.74 | 33% | 63% | 50% |
| Put Call Ratio (CBOE all options) | -1.03 | -1.51 | -1.10 | 58% | 63% | 46% |
| Inverse Relative U.S. Wealth | -0.86 | -2.33 | -0.49 | 58% | 63% | 33% |
| $D\beta_0(-1)$ | -1.72 | 2.54 | 0.83 | 58% | 63% | 42% |
| $D\beta_1(-1)$ | 1.02 | 0.88 | -1.67 | 46% | 63% | 58% |
| $D\beta_2(-1)$ | -0.33 | 1.39 | 1.36 | 58% | 58% | 58% |

is not likely to be the best specification. That is, it is possible that a non-linear model involving more than one of the variables will turn out to have significant predictive power. Also, predictability should be tested on an out-of-sample basis, with a process focused on finding the best possible trade-off between quality of fit and robustness.

Given the wide range of filters applied to select factors and models, there is of course a potential concern over the pitfalls of data mining. We try to mitigate this problem by using a *recursive modeling* and *thick* approach. The recursive modeling approach uses a three-stage procedure involving a calibration period, a training period, and a trading period. This procedure, suggested, for example, by Pesaran and Timmermann [1995], directly relates to the critique made by Bossaerts and Hillion [1999], who show the insufficiency of in-sample criteria to forecast out-of-sample information ratios.

For example, for a forecast starting in September 2000, we first decompose the six-year period September 1994 through August 2000 into two subperiods, a calibration period and a training period. In the *calibration period*, we use a four-year rolling window of data (starting in September 1994) to calibrate the model, i.e., estimate the coefficients. For the *training period*, we use a two-year rolling window of data (starting in September 1998) to backtest the model, i.e., generate forecasts and compute hit rates. Finally, we select the model at the end of the training period and use it subsequently in the three-year *trading period* (September 2000 to September 2003).

We actually extend the Pesaran and Timmermann [1995] recursive modeling approach to account for model uncertainty. They select in each period only one forecast, the forecast generated by the best model selected on the basis of a specified selection criterion (such as adjusted R-square, BIC, Akaike, or Schwartz) that weights goodness of fit against parsimony of the specification. We follow Granger and Jeon [2004] and label this approach "thin" modeling in that the forecasts of excess returns and consequently the performance of the asset allocation strategy are described over time by a thin line.

One limit of thin modeling is that model uncertainty is not considered. In each period, the information coming from the discarded models is ignored for the forecasting and portfolio allocation exercise. Focusing on a single predictive model may not be optimal, according to recent research along Bayesian lines, which stresses the importance of the estimation risk for portfolio allocation (see, for example, Barberis [2000] or Kandel and Stambaugh [1996]).

A natural way to interpret model uncertainty is to refrain from assuming there is a *true* model and instead attach probabilities to different possible models. This approach has been labeled Bayesian model averaging (see, for example, Avramov [2002] or Cremers [2002]). Bayesian methodology reveals the possibility of in-sample and out-of-sample predictability of stock returns, even when commonly adopted model selection criteria fail to demonstrate out-of-sample predictability.

The main difficulty with the application of Bayesian model averaging to problems like ours lies in specification of prior distributions for the parameters in all possible models of interest. Doppelhofer, Miller, and Sala-i-Martin [2004] have recently proposed an approach labeled Bayesian averaging of classical estimates (BACE) that overcomes the need to specify priors by combining the averaging of estimates across models, a Bayesian concept, with classic OLS estimation, interpretable in the Bayesian camp as coming from the assumption of diffuse, non-informative, priors.[9]

In a related line of research, Aiolfi and Favero [2002] argue that portfolio allocation strategies based on a thick modeling strategy (i.e., averaging across the different portfolio choices driven by predictions of excess returns) systematically outperform portfolio allocation strategies based on thin modeling. We apply the BACE approach by selecting at each date a "council" of models to make predictions rather than using a single model. Most long-short managers could use a similar methodology to enhance the performance of their portfolios without having to rely on the alleged superior performance of any specific predictive model.

To forecast changes in beta parameters, we use the 12 variables in Exhibit 6 and lagged values of the beta parameters. We test the explanatory power of not only the one-month lag $X_{t-1}$, but also of the squared lag $X_{t-1}^2$ (a measure of volatility), relative changes $\log(X_{t-1}/X_{t-2})$ (when relevant, i.e., when the variable is not already expressed as a return), and absolute changes $X_{t-1} - X_{t-2}$.

The next step is to select a set of models to forecast changes in the beta parameters. The process is based on two types of indicators. Indicators of type 1 are meant to represent the in-sample performance of the forecasting model, measured in terms of t-statistics and the Schwartz information criterion (SIC). The SIC lets us handicap the different models for the number of degrees of freedom more severely than by the adjusted R-square measure. To increase our confidence in the model's robustness, we do not consider models with more than four variables.[10]

Indicators of type 2 are meant to represent the out-of-sample forecasting power measured in terms of hit rate (accuracy of the direction).

During the trading period, we allow for a dynamic updating procedure. On each date, we select a group of models according to criteria as follows: 1) all variables in the model are significant at the 5% confidence level; 2) variables have been significant at the 5% level in 95% of the previous 12 months; and 3) hit rates in the training sample are higher than 0.55.

Criterion (1) ensures we are selecting a valid model; criterion (2) ensures that the model has shown robustness through time; and criterion (3) ensures that the model has demonstrated some minimum level of correct forecasting. A last step is to eliminate redundant models. More specifically, we do not let models that show 100% agreement to be part of the same "council."

Using a normality assumption for the residuals of the OLS regressions, we estimate the probability $p$ that changes in beta parameters are positive. In a thick modeling approach, one is left with $n$ potentially conflicting predictions at each date. We denote by $p_i$ the predicted probability for a positive change (increase) in a given beta parameter for model $i$.

Two important quantities of interest are the average forecast probability:

$$m_p = \frac{1}{n}\sum_{i=1}^{n} w_i p_i$$

and the standard deviation of the forecast:

$$\sigma_p = \sqrt{\frac{1}{n}\sum_{i=1}^{n} w_i\left(m_p - p_i\right)^2}$$

where $w_i$ is the weight associated with model $i$. This weight can be a function of the model's perceived ability to forecast. Given that no obviously relevant weighting scheme is available in our context, since the filter we have applied implies a relative level of homogeneity in the models' (in-sample measures of) performance, we set this weight equal to $1/n$.

The prediction rule is as follows. When $m_p$ exceeds 50% (the neutral view), this means that on average the models in the council predict the beta parameter to increase in value (positive change). We take the confidence in the prediction to be a function of how far above or below the neutral value of 50% the average $m_p$ is. We distinguish between two results: cases when the average forecast prob-

## EXHIBIT 7
**Outcome of Predictive Models**

| | | | $\beta_1$ | | |
|---|---|---|---|---|---|
| Number of Stand. Dev. away from Neutral View | 0 | 0.5 | 1 | 1.5 | 2 |
| Number of Bets | 43 | 34 | 27 | 14 | 9 |
| Hit Rate | 67% | 68% | 67% | 71% | 67% |
| | | | $\beta_2$ | | |
| Number of Stand. Dev. away from Neutral View | 0 | 0.5 | 1 | 1.5 | 2 |
| Number of Bets | 37 | 26 | 16 | 9 | 7 |
| Hit Rate | 54% | 58% | 69% | 56% | 71% |

ability is more than one standard deviation away from 50% (lower confidence in the forecast), and cases when the average forecast probability is less than one standard deviation away from 50% (higher confidence in the forecast).

The results for the out-of-sample period September 2000 to September 2003 are summarized in Exhibit 7.

Given our set of filters, it actually proves impossible over the sample period to calibrate any satisfactory model for changes in the level of interest rates ($\beta_0$ parameter), again reinforcing the impression that changes in interest rates are not predictable at the monthly level. Yet rather satisfactory results are obtained for other dimensions of the shape of the yield curve. That is, in the case of predictions of the slope of the yield curve ($\beta_1$), hit rates are always higher than two-thirds, whatever the number of standard deviations away from the consensus we consider. These numbers are significantly higher than 50% (null hypothesis of no predictability) at the 2.5% confidence level when there are at least 24 observations.

## III. IMPLEMENTING SYSTEMATIC TRADING STRATEGY

We can exploit this evidence in the level factor in active portfolio strategies implemented through trading in fixed-income derivatives. We use forecast changes in beta parameters to implement a systematic trading strategy using five standard swap butterflies. We first explain how we build swap butterflies, and then the trading rule used to position them, and finally we discuss the results.

### Butterfly Strategies

Bond or swap butterflies are among the most common active strategies that practitioners use to exploit views on interest rate changes.

A swap butterfly is a combination of short- and long-term plain vanilla swaps (called the *wings*) and a medium-term swap (called the *body*). In a *receiver swap butterfly*, the investor receives the fixed leg of the body and pays the fixed leg of the wings, while the opposite holds in a *payer swap butterfly*.[11] Because a plain vanilla swap is simply a bond minus a nominal amount, swap butterflies are built just like bond butterflies (see Martellini, Priaulet, and Priaulet [2003]). The advantage of swap butterflies is that they are always cash-neutral, which is not necessarily the case with bond butterflies.[12]

In an attempt to bet on specific views on changes of the shape (or slope and curvature) of the term structure, one wants to make the butterfly insensitive to the level and slope factors while keeping it exposed to changes in the curvature factor, or make it insensitive to the level and curvature factors while keeping it exposed to changes in the slope factor.

If we try to bet on a change in the curvature factor, portfolio weights are obtained as the solution to:

$$\begin{cases} q_s D_s L_s + q_l D_l L_l + \alpha D_m L_m = 0 \\ q_s D_s S_s + q_l D_l S_l + \alpha D_m S_m = 0 \end{cases}$$

where:

$q_s$, $\alpha$, and $q_l$ are the principal amounts of short-, medium-, and long-term swaps;
$D_s$, $D_m$, and $D_l$ are the modified durations of short-, medium-, and long-term swaps;
$L_s$, $L_m$, and $L_l$ are the sensitivity of short-, medium-, and long-term swap rates to the coefficient $\beta_0$; and
$S_S$, $S_m$, and $S_l$ are the sensitivity of the short-, medium-, and long-term swap rates to the coefficient $\beta_1$.

Since $L_S = L_m = L_l$ by construction, hedging against the level factor is equivalent to the duration-neutral condition, and the problem simplifies to

$$\begin{cases} q_s D_s + q_l D_l + \alpha D_m = 0 \\ q_s D_s = -\alpha D_m \gamma \end{cases}$$

where:

$$\gamma = \left( \frac{S_l - S_m}{S_l - S_s} \right)$$

Each month the sensitivities of these swap rates are calculated to the first and second factors $\beta_0$ and $\beta_1$, and for each swap butterfly combination the sensitivities to these beta parameters are derived, according to Martellini, Priaulet, and Priaulet [2003].

For a Nelson-Siegel-weighted payer swap butterfly, the total return in basis points is approximated by:[13]

$$\textit{Total return in bp} \approx \frac{\textit{Total return in \$}}{\alpha} \quad (4)$$

or

$$\textit{Total return in bp} \approx D_m \Delta r_m - \left( \frac{q_s D_s \Delta r_s + q_l D_l \Delta r_l}{\alpha} \right) + \text{Carry} \quad (5)$$

where:

$\Delta r_m$ = (swap rate of the swap with medium-term maturity − 1 month at date $t$ + 1 month) − (swap rate of the swap with medium-term maturity at date $t$).

Equation (5) simplifies into:

$$\textit{Total return in bp} = D_m \left[ \Delta r_m - \gamma \Delta r_s - (1 - \gamma) \Delta r_l \right] + \text{Carry} \quad (6)$$

Considering a 2/5/30 swap butterfly, $\Delta r_5$ would be the difference between the swap rate of the swap with maturity 59 months at date $t$ + 1 month and the swap rate of the swap with maturity 5 years at date $t$.

As positions are held during a period of one month, and as we position the same number of butterfly payers of fixed and butterfly receivers of fixed, which certainly creates compensation in terms of carry, we consider carry to be a negligible quantity from now on.

It is common practice in the market to consider the spread performance in basis points, the total return in basis points divided by the modified duration of the body:

$$\textit{Spread performance in bp} = \left[ \Delta r_m - \gamma \Delta r_s - (1 - \gamma) \Delta r_l \right]$$

As an illustration, consider a EUR 2-5-7 year combination on June 1, 2004. The coefficient $\gamma$ of this combination is equal to 0.334. The nominal amounts are 783, 1,000, and 499, respectively, on the short, medium, and long maturities. This has to be compared with a coefficient $\gamma$ equal to 0.5 for a 50-50 weighting, and weights equal to 1,171, 1,000, and 375.

Note also that another kind of Nelson-Siegel-weighted swap butterfly can be designed that is insensitive to changes in the level and curvature factors $\beta_0$ and $\beta_2$, while intentionally exposed to changes in the slope factor $\beta_1$. This can be achieved as before, and we obtain the same expression for total return and spread performance in basis points for this structure, except that we use a different $\gamma$ given by:

$$\gamma = \left( \frac{C_l - C_m}{C_l - C_s} \right)$$

where $C_S$, $C_m$, and $C_l$ are the sensitivity of the short-, medium-, and long-term swap rates to the coefficient $\beta_2$.

### Trading Rule

We consider the five standard 2-5-10 year, 2-5-30 year, 2-10-30 year, 5-10-15 year, and 5-10-30 year swap butterflies. When we bet on a move of $\beta_1$ ($\beta_2$) parameter, we consider the butterfly insensitive to $\beta_0$ and $\beta_2$ ($\beta_1$). The products have sensitivity to $\beta_1$($\beta_2$) that is constant over time. Exhibit 8 displays the sensitivities of these five butterflies to $\beta_1$ and $\beta_2$.

When we forecast an increase (decline) of the $\beta_1$ or $\beta_2$ parameter, we implement the payer (receiver) swap butterfly if the sensitivity is positive, and the opposite if the sensitivity is negative. Positions are held during a period of one month.

Finally, we take into account transaction costs, assumed to represent 0.5 basis points of the spread performance of a butterfly.

# EXHIBIT 8
**Sensitivities of Different Swap Butterflies**

| | 2/5/10 | 2/5/30 | 2/10/30 | 5/10/15 | 5/10/30 |
|---|---|---|---|---|---|
| Sensitivity to $\beta_1$ | -0.6830 | 0.7210 | -0.0772 | -0.6415 | -0.1110 |
| Sensitivity to $\beta_2$ | 0.1263 | 0.0608 | 0.0284 | 0.1187 | 0.0568 |

### Results

We use the methodology to implement systematic trading rules based on the signals generated by the econometric process. The cumulative spread performance of the five butterflies when betting either on the $\beta_1$ or $\beta_2$ parameter is displayed in Exhibits 9 and 10. These results show that predictability in the shape of the yield curve is both statistically and economically significant.

The best results are obtained with the 2-5-10 year, 2-5-30 year, and 2-10-30 year combinations, which exhibit cumulative spread performance of 278 bp (30 bp), 259 bp (50 bp), and 245 bp (37 bp) in betting on the $\beta_1$ ($\beta_2$) parameter when bets are taken when predicted value is one standard deviation away from the neutral view. These results suggest that more significant outperformance is obtained in betting on the slope factor instead of the curvature factor.

Interestingly, as can be seen in Exhibits 11 and 12, when there is more confidence in the prediction (as measured by the number of standard deviations from the neutral view), the percentage of trades with positive performance also increases (for example, from 60% to 75% for the 5-10-15 year butterfly as the number of standard deviations away from the neutral view increases from zero to two and in betting on $\beta_1$; see Exhibit 11). On the other hand, the cumulative spread is reduced because there are fewer trades initiated.

Another useful way to illustrate the benefits of such active strategies is to examine the risk-return patterns they generate. Exhibits 13 and 14 report standard measures of risk-adjusted performance (annualized return, annualized volatility, Sharpe and Sortino ratios, downside deviation) in the case of a bet on changes in $\beta_1$ and changes in $\beta_2$. While we have focused on a strategy that generates a trade even when the predicted probability is arbitrarily close to the neutral 50% reference, we also tested different levels of (net) leverage, 2, 4, and 20 (a (net) leverage equal to 4 means that an initial $100 is invested in cash, while another $200 is invested in a long or short position in the body of the butterfly, covered by a position short or long in the wings of the butterfly designed to ensure duration-neutrality).

Exhibit 13 shows spectacular performance of an active strategy based on bets on the $\beta_1$ parameter, with Sharpe ratios close to 2 and Sortino ratios above 3, whatever the particular level of leverage. On the other hand, as could be expected from results reported in Exhibit 12, bets on the curvature factor do not allow for significant outperformance (*see Exhibit 14*).

## IV. CONCLUSIONS

We have presented evidence of predictability in the time-varying shape of the U.S. term structure of interest rates. We find we can use variables such as default spread, equity volatility, and short-term and forward rates, among others, to predict changes in the slope of the yield curve (and to a lesser extent changes in the curvature of the yield curve). Systematic trading strategies based on butterfly swaps indicate that this evidence of predictability in the shape of the yield curve is both economically and statistically significant enough to be exploited.

# EXHIBIT 9
**Cumulative Spread Performance of Five Butterflies Betting on $\beta_1$**

| | Bets on $\beta_1$ | | | | |
|---|---|---|---|---|---|
| Number of Stand. Dev. away from Neutral View | 0 | 0.5 | 1 | 1.5 | 2 |
| Number of Bets | 43 | 34 | 27 | 14 | 9 |
| Hit Rate | 67% | 68% | 67% | 71% | 67% |
| | Cumulative spread performance | | | | |
| 2/5/10 | **430bp** | **358bp** | **265bp** | **148bp** | **75bp** |
| 2/5/30 | **375bp** | **316bp** | **246bp** | **145bp** | **72bp** |
| 2/10/30 | **366bp** | **307bp** | **232bp** | **133bp** | **67bp** |
| 5/10/15 | **30bp** | **24bp** | **15bp** | **2bp** | **1bp** |
| 5/10/30 | **59bp** | **48bp** | **31bp** | **15bp** | **8bp** |

## EXHIBIT 10
### Cumulative Spread Performance of Five Butterflies Betting on $\beta_2$

| | Bets on $\beta_2$ | | | | |
|---|---|---|---|---|---|
| Number of Stand. Dev. away from Neutral View | 0 | 0.5 | 1 | 1.5 | 2 |
| Number of Bets | 37 | 26 | 16 | 9 | 7 |
| Hit Rate | 54% | 58% | 69% | 56% | 71% |
| | Cumulative spread performance | | | | |
| 2/5/10 | **-10bp** | **-3bp** | **22bp** | **7bp** | **11bp** |
| 2/5/30 | **-6bp** | **1bp** | **42bp** | **17bp** | **28bp** |
| 2/10/30 | **-9bp** | **-6bp** | **29bp** | **14bp** | **28bp** |
| 5/10/15 | **-10bp** | **-10bp** | **-2bp** | **0bp** | **3bp** |
| 5/10/30 | **-15bp** | **-13bp** | **4bp** | **4bp** | **12bp** |

## EXHIBIT 11
### Percentage of Trades with Positive Spread Performance Betting on $\beta_1$

| | Bets on $\beta_1$ | | | | |
|---|---|---|---|---|---|
| Number of Stand. Dev. away from Neutral View | 0 | 0.5 | 1 | 1.5 | 2 |
| Number of Bets | 43 | 34 | 27 | 14 | 9 |
| Hit Rate | 67% | 68% | 67% | 71% | 67% |
| | % of trades with positive spread performance | | | | |
| 2/5/10 | 67% | 71% | 70% | 71% | **78%** |
| 2/5/30 | 63% | 65% | 67% | 64% | **67%** |
| 2/10/30 | 67% | 71% | 70% | 71% | **78%** |
| 5/10/15 | 60% | 68% | 67% | 64% | **78%** |
| 5/10/30 | 60% | 65% | 63% | 64% | **78%** |

## EXHIBIT 12
### Percentage of Trades with Positive Spread Performance Betting on $\beta_2$

| | Bets on $\beta_2$ | | | | |
|---|---|---|---|---|---|
| Number of Stand. Dev. Away from Neutral View | 0 | 0.5 | 1 | 1.5 | 2 |
| Number of Bets | 37 | 26 | 16 | 9 | 7 |
| Hit Rate | 54% | 58% | 69% | 56% | 71% |
| | % of trades with positive spread performance | | | | |
| 2/5/10 | 49% | 50% | 56% | 56% | 71% |
| 2/5/30 | 49% | 50% | 56% | 56% | 71% |
| 2/10/30 | 49% | 50% | 56% | 56% | 71% |
| 5/10/15 | 46% | 38% | 44% | 56% | 57% |
| 5/10/30 | 46% | 46% | 50% | 56% | 71% |

## EXHIBIT 13
**Risk-Return Profile of Active Strategies Based on Bet on $\beta_1$**

| Leverage 2 | Cash | 2y-5y-30y | 2y-5y-10y | 5y-10y-15y | 2y-10y-30y | 5y-10y-30y |
|---|---|---|---|---|---|---|
| Annual Return | 3.04% | **9.23%** | **9.89%** | **4.47%** | **13.17%** | **5.16%** |
| Annual Volatility | 0.56% | **3.68%** | **3.70%** | **1.07%** | **5.82%** | **1.32%** |
| Sharpe Ratio | | **1.7** | **1.8** | **1.3** | **1.7** | **1.6** |
| Downside Dev | | **2.01%** | **1.80%** | **0.60%** | **2.98%** | **0.64%** |
| Sortino Ratio | | **3.1** | **3.8** | **2.4** | **3.4** | **3.3** |

| Leverage 4 | Cash | 2y-5y-30y | 2y-5y-10y | 5y-10y-15y | 2y-10y-30y | 5y-10y-30y |
|---|---|---|---|---|---|---|
| Annual Return | 3.04% | **14.75%** | **16.06%** | **5.23%** | **22.63%** | **6.60%** |
| Annual Volatility | 0.56% | **7.17%** | **7.23%** | **1.80%** | **11.50%** | **2.36%** |
| Sharpe Ratio | | **1.6** | **1.8** | **1.2** | **1.7** | **1.5** |
| Downside Dev | | **3.86%** | **3.48%** | **0.99%** | **5.83%** | **1.12%** |
| Sortino Ratio | | **3.0** | **3.7** | **2.2** | **3.4** | **3.2** |

| Leverage 20 | Cash | 2y-5y-30y | 2y-5y-10y | 5y-10y-15y | 2y-10y-30y | 5y-10y-30y |
|---|---|---|---|---|---|---|
| Annual Return | 3.04% | **58.92%** | **65.46%** | **11.31%** | **98.29%** | **18.17%** |
| Annual Volatility | 0.56% | **35.31%** | **35.61%** | **8.23%** | **57.00%** | **11.10%** |
| Sharpe Ratio | | **1.6** | **1.8** | **1.0** | **1.7** | **1.4** |
| Downside Dev. | | **18.76%** | **17.04%** | **4.61%** | **28.75%** | **5.34%** |
| Sortino Ratio | | **3.0** | **3.7** | **1.8** | **3.3** | **2.8** |

## EXHIBIT 14
**Risk-Return Profile of Active Strategies Based on Bet on $\beta_2$**

| Leverage 2 | Cash | 2y-5y-30y | 2y-5y-10y | 5y-10y-15y | 2y-10y-30y | 5y-10y-30y |
|---|---|---|---|---|---|---|
| Annual Return | 3.04% | **2.97%** | **2.90%** | **2.78%** | **2.82%** | **2.65%** |
| Annual Volatility | 0.56% | **1.43%** | **0.97%** | **0.66%** | **2.02%** | **1.09%** |
| Sharpe Ratio | | **-0.05** | **-0.15** | **-0.40** | **-0.11** | **-0.35** |
| Downside Dev. | | **1.18%** | **0.87%** | **0.62%** | **1.54%** | **0.90%** |
| Sortino Ratio | | **-0.06** | **-0.17** | **-0.42** | **-0.14** | **-0.43** |

| Leverage 4 | Cash | 2y-5y-30y | 2y-5y-10y | 5y-10y-15y | 2y-10y-30y | 5y-10y-30y |
|---|---|---|---|---|---|---|
| Annual Return | 3.04% | **2.91%** | **2.76%** | **2.51%** | **2.64%** | **2.27%** |
| Annual Volatility | 0.56% | **2.69%** | **1.67%** | **1.07%** | **3.95%** | **1.99%** |
| Sharpe Ratio | | **-0.05** | **-0.17** | **-0.49** | **-0.10** | **-0.38** |
| Downside Dev. | | **2.07%** | **1.41%** | **0.93%** | **2.84%** | **1.55%** |
| Sortino Ratio | | **-0.06** | **-0.20** | **-0.57** | **-0.14** | **-0.50** |

| Leverage 20 | Cash | 2y-5y-30y | 2y-5y-10y | 5y-10y-15y | 2y-10y-30y | 5y-10y-30y |
|---|---|---|---|---|---|---|
| Annual Return | 3.04% | **3.13%** | **1.86%** | **0.45%** | **2.76%** | **-0.56%** |
| Annual Volatility | 0.56% | **13.45%** | **8.00%** | **5.67%** | **21.06%** | **10.38%** |
| Sharpe Ratio | | **0.01** | **-0.15** | **-0.46** | **-0.01** | **-0.35** |
| Downside Dev. | | **9.32%** | **5.94%** | **4.24%** | **13.89%** | **7.34%** |
| Sortino Ratio | | **0.01** | **-0.20** | **-0.61** | **-0.02** | **-0.49** |

## ENDNOTES

The authors thank Ludovic Breger, Mehdi Chaabouni, Oren Cheyette, Martijn Cremers, and Wesley Phoa for helpful comments and suggestions and Luc Hazoume for help with the numerical tests. The views expressed here are those of the authors themselves and should in no way be attributed to their employers.

[1]See also Duffee [2002] for an attempt to generate forecasts of future changes in the level of interest rates using standard affine term structure models.

[2]See Martellini and Priaulet [2000] for details.

[3]While the Nelson-Siegel model is based on constant parameters, we use a rolling window analysis to fit the model and extract a dynamic view of term structure deformations. Such a procedure, while formally inconsistent, is widely used in practice, and is similar to an estimate of the time series of implied volatility from the Black-Scholes model of option prices, even though the model is based on the assumption of a constant volatility.

[4]Following common practice, we hold $\tau$ constant in the analysis (see Barrett, Gosnell, and Heuson [1995] and Willner [1996]). We could, of course, take it as an additional variable, but by opting for such a solution, we might create instability in the beta parameters.

[5]Because we subsequently focus on swap butterflies, as opposed to bond butterflies, we model swap rates instead of zero-coupon yields like Nelson and Siegel [1987].

[6]Hit rate is defined as the percentage of times the correct sign of changes in the parameter is predicted.

[7]Given that market participants use forward rates as an indication of investor expectations concerning future movements of the short-term rate, a model's ambition should be not only to predict future changes in interest rate level, slope, or curvature, but also to outperform predictions of simple models based on forward rates only.

[8]This is a priori better than including a measure of inflation, which is by construction a backward-looking estimate.

[9]As an alternative, Cremers [2002] suggests using economically meaningful prior information such as some prior sense of the $R^2$ of the predictive regression and variance of the residuals and number of predictors. While intuitively appealing, this methodology results in priors for model parameters that are relatively flat, so that posterior estimates will be dominated by sample data as in the simpler procedure that we follow.

[10]We also check to ensure multicollinearity is not an issue, as the maximum correlation between two variables does not exceed 60% in the sample.

[11]In the U.K., the receiver swap butterfly is called a *barbell.*

[12]Bond butterflies actually come in different forms: 50/50, maturity weighting, minimum variance weighting, regression weighting, PCA weighting, and Nelson-Siegel weighting.

[13]To be more precise, we would have to incorporate the convexity term, which is omitted here as positions are carried out over a period of one month, and small changes in yields are assumed.

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